math eq:

① $\int_a^b dx = x \Big|_a^b$ ② $\int_a^b \frac{1}{x} dx = \ln x \Big|_a^b$ ③ $\int_a^b \frac{1}{x^2} dx = -\frac{1}{x} \Big|_a^b$

Integrated Rate Law

0th order

$$-\frac{d[A]}{dt} = k \Rightarrow d[A] = -k\,dt$$

$$\int_{A_0}^{A_t} d[A] = -k \int_0^t dt$$

Note: A_t: concentration of A at time t

A_0: concentration of A at time o

use math eq #1

$$[A]\Big|_{A_0}^{A_t} = -k\,t\Big|_0^t \Rightarrow [A]_t - [A]_0 = -k(t-0)$$

$$\boxed{[A]_t = [A]_0 - kt}$$

1st order

$$-\frac{d[A]}{dt} = k[A] \Rightarrow \frac{d[A]}{[A]} = -k\,dt \Rightarrow \int_{A_0}^{A_t} \frac{1}{[A]} d[A] = -k \int_0^t dt$$

use math eq 2

$$\ln[A]\Big|_{A_0}^{A_t} = -k\,t\Big|_0^t$$

$$\boxed{\ln \frac{[A]_t}{[A]_0} = -kt}$$

$$\ln[A]_t - \ln[A]_0 = -kt$$

$$\Rightarrow \boxed{\ln[A]_t = \ln[A_0] - kt}$$

take exp on both sides

$$\frac{[A]_t}{[A_0]} = \exp(-kt)$$

$$\boxed{[A]_t = [A]_0 \exp(-kt)}$$

2nd order

$$-\frac{d[A]}{dt} = k[A]^2 \Rightarrow \frac{d[A]}{[A]^2} = -k\,dt \Rightarrow \int_{A_0}^{A_t} \frac{1}{[A]^2} d[A] = -k \int_0^t dt$$

use math eq 3

$$-\frac{1}{[A]}\Big|_{A_0}^{A_t} = -k\,t\Big|_0^t$$

canceal out

$$\left(\frac{1}{[A]_t} - \frac{1}{[A]_0}\right) = kt$$

$$\boxed{\frac{1}{[A]_t} = \frac{1}{[A]_0} + kt}$$